ELASTOPLASTICITY AND THE ATTENUATION OF SHOCK WAVES

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Calculations have been performed for the case of a thin projectile striking a semi-infinite target. Because the projectile is considered to be infinite in extent in directions perpendicular to its direction of travel, the resulting flow is one-dimensional in the strain. It is assumed that stresses and strains are related by elastoplastic relations. Work of other authors has been extended so that (1) the shear modulus depends on the magnitude of the hydrostatic compression; (2) the yield is increased as the hydrostatic pressure is increased; (3) the artificial viscosity method is used for solving the flow equations. Poisson's ratio is assumed to be a constant. Comparison of the calculated results with experimental results gives strong support to the assumptions that both aluminum and copper yield elastoplastically when the maximum stress is about 0.1 megabars. No difficulties are found in applying the artificial viscosity method developed by von Neumann and Richtmyer to problems involving elastoplastic flow.

I. Introduction

The mechanics of attenuation of stress waves in elastoplastic solids has been the subject of extensive theoretical and experimental investigations during the last twenty-five years. Most of these studies have been directed toward the understanding of wave propagation in rods and, after the first blush of success reported by von Karman and Taylor, successful correlations of calculations and measurements have been few. At least part of this failure has been due to the essentially two-dimensional character of wave propagation on a bar; other deviations have been widely attributed to the dependence of yield point and plastic moduli on strain rates.

In recent years it has become possible to produce well-controlled plane waves in solids by means of explosive or high velocity impact. During the same period techniques for precisely measuring the waves so produced have been developed, and meaningful comparisons between measurements and theoretical calculations under conditions of true uniaxial strain have become possible. During this same period the development of high speed computing machines has made it a relatively simple matter to systematically alter the constitutive relations used for wave calculations and so to approach a theoretical model that agrees arbitrarily well with experimental measurements. It was toward such a comparison that work reported here was directed.

The work was performed in a study of the attenuation of shock waves in aluminum and copper. The amplitude of the stress carried by the shock waves was about 0.1 megabar (10^{11} dynes/cm^2, or approximately 10^5 atmospheres). Measurements were made of the free-surface velocities of targets that had been hit by projectiles in the form of 1/8 inch thick aluminum plates that had been accelerated to a velocity of about 0.125 cm/μsec by high explosives. The use of targets of progressively greater thickness gave information on the attenuation of the shock in the material being studied.

The results of these experiments are compared with the results of calculations based on an elastoplastic model for the relation of stress to strain (or stress to volume) and a fluid model.

The compression in a uniform shock wave traveling through an elastoplastic medium can be adequately described with conventional analysis supplemented by relatively simple numerical computations based on the theory of characteristics. When the shock wave is nonuniform, wave interactions complicate the calculations, as do the presence of interfaces between materials having different acoustic impedances. For such reasons, the method of characteristics is not suitable in many situations for which numerical solutions are desired. An alternative is available in the form of the method of von Neumann and Richtmyer [1]. Boundaries, including free surfaces, are readily accommodated by this method and its application to the flow in an elastoplastic medium is straightforward. A detailed comparison of the relative advantages of the theory of characteristics and the method of von Neumann and Richtmyer (hereafter called the Q-method) in different problems is given by Fife, Eng, and Young [2] for the simpler case of wave propagation in a fluid.

The Q-method consists of using finite difference equations based on increments in real space and time to replace the differential equations that describe the flow, and of smoothing discontinuities through introduction of an artificial viscosity. For one-dimensional flow, the equations of continuity and motion in Lagrangian coordinates are

$$\rho_0 \frac{\partial V}{\partial t} = \frac{\partial u}{\partial x} \tag{1}$$

$$\rho_0 \frac{\partial u}{\partial t} = - \frac{\partial (p_x + Q)}{\partial x}, \tag{2}$$

where u is particle velocity, V is specific volume, t is time, x is Lagrange space coordinate, ρ_0 is density of the undisturbed medium, and p_x is compressive stress in the x direction. Q is the artificial viscosity. The particular form for Q used in the present case is

$$Q = - \frac{(C_q \Delta x)^2}{V} \frac{\partial u}{\partial x} \left| \frac{\partial u}{\partial x} \right|. \tag{3}$$

This form for Q satisfies the following conditions:

a. It eliminates discontinuities in the flow field;

b. The thickness of the shock layer is the order of the space increment, Δx, used in the computation;

c. The effects of Q outside the shock layers are negligible;

d. The Rankine-Hugoniot equations hold if gradients outside the shock layer are small.

The use of this method when p_x and V are related by an elastoplastic model has caused no problems with instabilities in the calculations.

II. The Elastoplastic Relation Between Stress and Strain

Earlier work on the theory of plane wave propagation in elastoplastic materials was done by White and Griffis [3], Wood [4], and Morland [5]. Their numerical methods were inadequate for generalization to finite amplitude waves, however, and this work may be regarded as an extension of theirs. At present only one-dimensional strain is considered, and the direction of propagation is taken to be the x-axis; stresses and strains without subscripts relate to that direction. Figure 1 shows the stress-strain relations in a cycle of one-dimensional compression starting at $p_x = \eta = 0$. We can always write the identity

$$dp_x = d\bar{p} + 4/3\, d\tau, \tag{4}$$

where $\bar{p}$ is mean compressive stress, p_x is compressive stress in the direction of propagation, and $d\tau = (dp_x - dp_y)/2$ is maximum resolved shear stress. In the plastic state, $d\tau = 0$ except for strain hardening. For both the Tresca and the von Mises yield conditions in this geometry, the maximum value of τ is determined by the yield stress, $Y = 2\tau$. Equation (4) is to be specialized for the curve segments *ba, ae, ef,* and *fb* of Fig. 1

Along *ba:*

The material is behaving elastically, $\tau < Y/2$, and so

$$dp_x = (K + 4\mu/3) d\epsilon_x = 3d\bar{p}(1 - \nu)/(1 + \nu), \tag{5}$$